Holographic superconductors in the AdS black-hole spacetime with a global monopole

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Abstract

We study holographic superconductors in the Schwarzschild-AdS black hole with a global monopole through a charged complex scalar field. We calculate the condensates of the charged operators in the dual conformal field theories (CFTs) and discuss the effects of the global monopole on the condensation formation. Moreover, we compute the electric conductive using the probe approximation and find that the properties of the conductive are quite similar to those in the Schwarzschild-AdS black hole. These results can help us know more about holographic superconductors in the asymptotic AdS black holes.

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I. INTRODUCTION

One of the most significant discovery in the string theory is the AdS/CFT correspondence which proposed by Maldacena [1] in the last century. It tells us that the gravity theory in a $(d+1)$-dimensional AdS spacetime can be described by a conformal field theory on the d-dimensional boundary. The AdS/CFT correspondence is a powerful tool to understand the strongly coupled gauge theories [2–4]. Recently, it has been applied extensively to investigate some condensed matter phenomena, such as superconductivity [5–8], superfluid [9, 10], and so on.

Superconductivity is a common phenomenon occurring in certain materials at very low temperatures. The most striking feature of the phenomenon is that the material has exactly zero electrical resistance for a direct current (DC) and the exclusion of the interior magnetic field when it is in the superconducting state. The first model for the holographic superconductors in the AdS black-hole spacetime is proposed by Hartnoll, Herzog and Horowitz [5]. They considered the classical instability of a black hole in AdS spacetime against perturbation by a charged scalar field and found that if the temperature T of the black hole is below a critical temperature T_c the instability of the black hole emerges, which implies that the original Schwarzschild-AdS black hole should be replaced by a new black hole with the charged scalar hair. According to the AdS/CFT correspondence, the emergence of the hairy AdS black hole means the formation of a charged condensation in the dual CFTs [11]. Moreover, the expectation values of the charged operators undergo the U(1) symmetry breaking and then the superconductive is formed.

Hartnoll, Herzog and Horowitz [5] also calculated the electrical conductive of the charged condensation and found that the condensation has zero electrical DC resistance, which is the same as that obtained in the Bardeen-Cooper-Schrieffer (BCS) theory [12]. This has triggered many people to study holographic superconductors in the various theories of gravity [13–38]. Gregory *et al* [13, 19] considered the holographic superconductors in the Einstein-Gauss-Bonnet gravity and found that the higher curvature corrections make condensation harder to form. Cai *et al* [20–22] investigated the holographic superconductor models in the Hořava-Lifshitz gravity. Recent studies also showed that there exist holographic superconductors in the string/M theory [23–27]. The properties of holographic superconductors at the zero-temperature limit have also been considered extensively [28–31]. These results can help us to understand more about the holographic superconductors in the asymptotical AdS black holes.

In this paper, we will investigate the holographic superconductors in planar AdS black hole with a global

monopole. A global monopole is one of the topological defects which could be formed during phase transitions in the evolution of the early Universe. The metric of the black hole with a global monopole was obtained by Barriola and Vilenkin [39], which arises from the breaking of global $SO(3)$ symmetry of a triplet scalar field in a Schwarzschild background. The presence of the global monopole results in that the black hole has different topological structure from that of the Schwarzschild black hole. The physical properties of the black hole with a global monopole have been studied extensively in recent years [40–43]. The main purpose in this paper is to see how the global monopole affects the holographic superconductors in this asymptotic AdS black hole.

This paper is organized as follows. In Sec. II, we present the metric describing a planar Schwarzschild-AdS black hole with a global monopole. In Sec. III, we give the basic equations and study numerically holographic superconductors in the planar black hole background with a global monopole. Our results show that the global monopole presents the different effects on the different condensations. In Sec. IV, we calculate the electrical conductivity of the charged condensates by ignoring the backreaction of the dynamical matter field on the spacetime metric. Finally, in the last section we present our conclusions.

II. THE PLANAR SCHWARZSCHILD-ADS BLACK HOLE WITH A GLOBAL MONOPOLE

The simplest model that gives rise to the global monopole is described by the Lagrangian

$$\mathcal{L}_{Gm} = \frac{1}{2}\partial_\mu \Phi^a \partial^\mu \Phi^{*a} - \frac{\gamma}{4}(\Phi^a \Phi^{*a} - \eta^2)^2, \tag{1}$$

where Φ^a is a multiplet of scalar field, η is the energy scale of symmetry breaking and γ is a constant. The metric ansatz describing a planar black hole with a global monopole can be taken as

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(dx^2 + dy^2). \tag{2}$$

In this case the field configuration describing a monopole is

$$\Phi^1 = \eta h(r)e^{ix}\cos y, \quad \Phi^2 = \eta h(r)e^{ix}\sin y. \tag{3}$$

Using the Lagrangian (1) and metric (2), we can calculate the energy-momentum tensor

$$T_{\mu\nu} = 2\frac{\partial \mathcal{L}_{Gm}}{\partial g^{\mu\nu}} - \mathcal{L}_{Gm}g_{\mu\nu}, \tag{4}$$

and then obtain the nonzero components of the energy-momentum tensor

$$T_t^t = \frac{1}{2}\eta^2 h'(r)^2 f(r) + \eta^2 \frac{h(r)^2}{r^2} + \frac{\gamma}{4}\left[\eta^2 h(r)^2 - \eta^2\right]^2,\tag{5}$$

$$T_r^r = -\frac{1}{2}\eta^2 h'(r)^2 f(r) + \eta^2 \frac{h(r)^2}{r^2} + \frac{\gamma}{4}\left[\eta^2 h(r)^2 - \eta^2\right]^2,\tag{6}$$

$$T_x^x = T_y^y = \frac{1}{2}\eta^2 h'(r)^2 f(r) + \frac{\gamma}{4}\left[\eta^2 h(r)^2 - \eta^2\right]^2.\tag{7}$$

As in Ref. [39], we take an approximation $h(r) = 1$ outside the core. It is reasonable because that $h(r)$ grows linearly when $r < (\eta\sqrt{\gamma})^{-1}$ and approaches exponentially unity as soon as $r > (\eta\sqrt{\gamma})^{-1}$.

In the planar AdS background, the Einstein equations are given by

$$\frac{f(r)'}{r} + \frac{f(r)}{r^2} - \frac{3}{L} + \frac{8\pi\eta^2}{r^2} = 0,\tag{8}$$

$$\frac{f(r)''}{2} + \frac{f(r)'}{r} - \frac{3}{L} = 0,\tag{9}$$

where L is the radius of AdS. An exact solution of the equations (8) and (9) is

$$f(r) = \frac{r^2}{L^2} - \tilde{b} - \frac{2M}{r},\tag{10}$$

where the parameter $\tilde{b} = 8\pi\eta^2$. This solution describes a planar Schwarzschild-AdS black hole with a global monopole. As $\tilde{b}$ tends to zero, the spacetime reduces to a four dimensional Schwarzschild-AdS black hole. The Hawking temperature of the black hole (10) is

$$T_H = \frac{3r_H^2 - \tilde{b}}{4\pi r_H},\tag{11}$$

where r_H is the event horizon of the black hole and we set the AdS radius is $L = 1$.

III. THE CONDENSATE OF CHARGED OPERATORS

In order to investigate the holographic superconductors in the background of the planar Schwarzschild-AdS black hole with a global monopole, we need a condensate through a charged scalar field. Here we adopt to the probe approximation and neglect the backreaction of the charged scalar field on the background. It must be pointed out that this charged scalar field is not the multiplet scalar field describing the global monopole. It may be an interesting topic to study the holographic superconductors of the scalar field which gives rise to the global monopole. However, it is very difficult because that for the scalar field with the form (3) the coupled equation (15) cannot be separable in the following calculations. Thus, in this paper we only consider the condensate of an external charged scalar field in the background of a black hole with a global monopole.

As in Ref. [5], let us consider a Maxwell field and a charged complex scalar field. The Lagrangian can be expressed as [5]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |\nabla_\mu\psi - iqA_\mu\psi|^2 - m^2\psi^2, \tag{12}$$

where $F_{\mu\nu}$ is electromagnetic tensor and ψ is a charged complex scalar field. Adopting to the ansatz

$$A_\mu = (\phi(r), 0, 0, 0), \quad \psi = \psi(r), \tag{13}$$

we can obtain the equations of motion for the complex scalar field ψ and electrical scalar potential $\phi(r)$ in the background of the Schwarzschild-AdS black hole with a global monopole

$$\psi'' + (\frac{f'}{f} + \frac{2}{r})\psi' + \frac{q^2\phi^2\psi}{f^2} - \frac{m^2\psi}{f} = 0, \tag{14}$$

and

$$\phi'' + \frac{2}{r}\phi' - \frac{2\psi^2}{f}\phi = 0, \tag{15}$$

respectively. Here a prime denotes the derivative with respect to r. Obviously, it is very difficult to obtain the nontrivial analytical solutions to the nonlinear equations (14) and (15). So we have to resort numerical method to solve above equations. In general, the boundary condition on the scalar potential ϕ near the black hole horizon $r \sim r_H$ is imposed as $\phi = 0$ so that its finite norm can be satisfied. Combining with Eq. (14), it is easy to obtain that $\psi = -\frac{3r_H^2 - \tilde{b}}{2r_H}\psi'$, which means that the complex scalar field ψ is regular at $r = r_H$. At the spatial infinite $r \to \infty$, the scalar field ψ and the scalar potential ϕ can be approximated as

$$\psi = \frac{\psi^{(1)}}{r} + \frac{\psi^{(2)}}{r^2} + ..., \tag{16}$$

and

$$\phi = \mu - \frac{\rho}{r} + ... \tag{17}$$

According to the dual theory, the constants μ and ρ in the asymptotic form of ϕ are the chemical potential and the charge density on the boundary, respectively. The expectation values of the condensate operator $\mathcal{O}$ dual to the scalar field ψ. Since for ψ both of these falloffs are normalizable [44], one can impose the boundary condition either $\psi^{(1)} = 0$ or $\psi^{(2)} = 0$ to keep the theory stable in the asymptotic AdS region. Therefore, we have

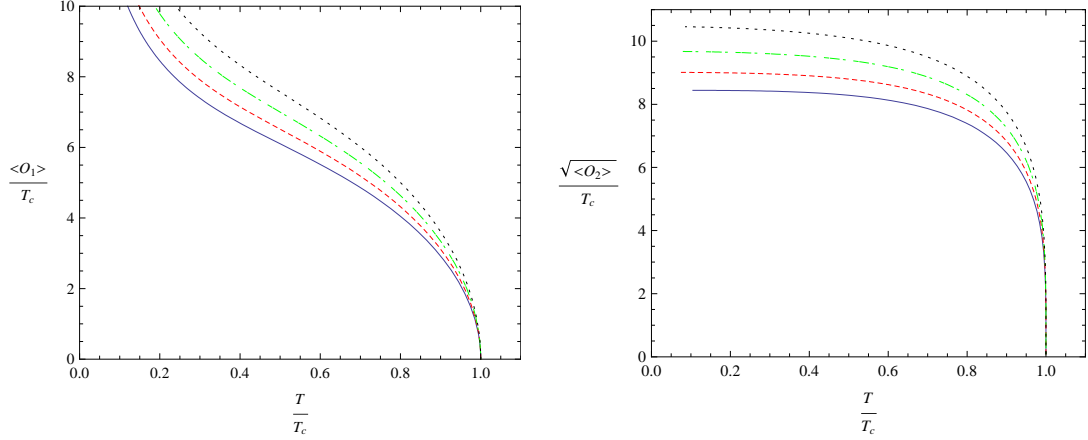$$\langle \mathcal{O}_1 \rangle = \sqrt{2}\psi^{(1)}, \quad \psi^{(2)} = 0, \tag{18}$$

FIG. 1: The condensates of operators $\mathcal{O}_1$(left) and $\mathcal{O}_2$(right) versus temperature. The condensates disappear as $T \to T_c$. The condensate is a function of temperature for various values of b. Here b is the re-scaled $\tilde{b}$ given by $b = \tilde{b}r_H^2/L^2$. The lowest solid line is $b = 0$, the dashed line is $b = 0.25$, the dash-dotted line is for $b = 0.5$, and the top dotted line is $b = 0.75$.

b	$\mathcal{O}_1$		$\mathcal{O}_2$	
0	$T_c \approx 0.226\rho^{1/2}$	$\langle\mathcal{O}_1\rangle \approx 9.30T_c(1 - T/T_c)^{1/2}$	$T_c \approx 0.118\rho^{1/2}$	$\langle\mathcal{O}_2\rangle \approx 144\, T_c^2(1 - T/T_c)^{1/2}$
0.25	$T_c \approx 0.222\rho^{1/2}$	$\langle\mathcal{O}_1\rangle \approx 9.95T_c(1 - T/T_c)^{1/2}$	$T_c \approx 0.111\rho^{1/2}$	$\langle\mathcal{O}_2\rangle \approx 155.0T_c^2(1 - T/T_c)^{1/2}$
0.50	$T_c \approx 0.223\rho^{1/2}$	$\langle\mathcal{O}_1\rangle \approx 10.5T_c(1 - T/T_c)^{1/2}$	$T_c \approx 0.103\rho^{1/2}$	$\langle\mathcal{O}_2\rangle \approx 174.24T_c^2(1 - T/T_c)^{1/2}$
0.75	$T_c \approx 0.238\rho^{1/2}$	$\langle\mathcal{O}_1\rangle \approx 11.4T_c(1 - T/T_c)^{1/2}$	$T_c \approx 0.096\rho^{1/2}$	$\langle\mathcal{O}_2\rangle \approx 196.56T_c^2(1 - T/T_c)^{1/2}$

TABLE I: The critical temperature and the expectation values for the operators $\mathcal{O}_1$ and $\mathcal{O}_2$ when $T \to T_c$ for different values of b.

or

$$\langle\mathcal{O}_2\rangle = \sqrt{2}\psi^{(2)}, \quad \psi^{(1)} = 0. \tag{19}$$

As in Refs. [5–8], the factor of $\sqrt{2}$ is a convenient normalization and the index i denotes the scaling dimension λ of its dual operator $\langle\mathcal{O}_i\rangle$.

In Fig.1 we plot the condensates of operators $\mathcal{O}_1$ and $\mathcal{O}_2$ for different b, which is the re-scaled $\tilde{b}$ given by $b = \tilde{b}r_H^2/L^2$. We find that if the temperature T of the black hole is below the critical temperature T_c, the expectation values of the condensate operators $\mathcal{O}_1$ and $\mathcal{O}_2$ tend to the constants for fixed b. It is qualitatively similar to that obtain in BCS theory [12]. This means that there exists the holographic superconductors in the Schwarzschild-AdS black hole with a global monopole. As in Refs. [5–8], the expectation value $\langle\mathcal{O}_1\rangle$ for
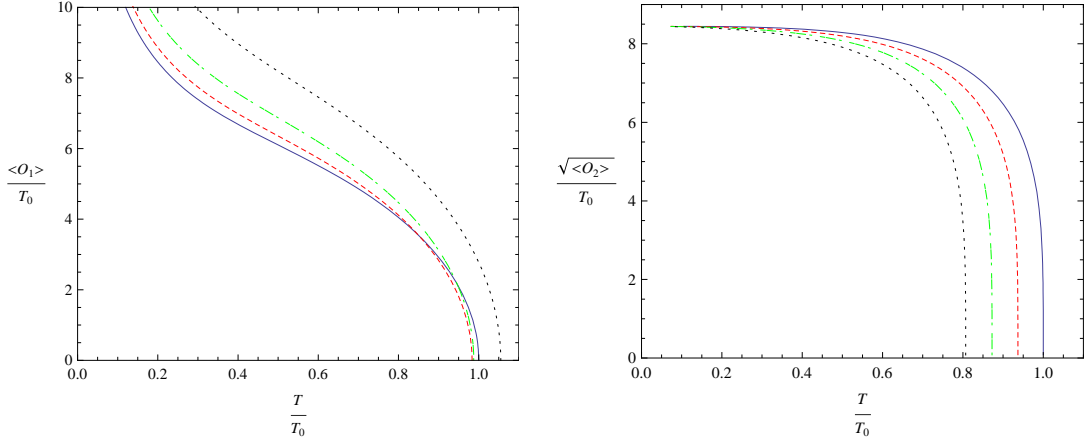
FIG. 2: The condensates of operators $\mathcal{O}_1$(left) and $\mathcal{O}_2$(right) versus temperature. The condensate is a function of temperature for various values of b. T_0 is the critical temperature T_c at $b = 0$. The solid line is $b = 0$, the dashed line is $b = 0.25$, the dash-dotted line is for $b = 0.5$, and the dotted line is $b = 0.75$. The critical temperature for $\mathcal{O}_2$ decreases as b increases, while for $\mathcal{O}_1$ it first decreases and then increases. At very low temperature, the expectation values $\langle\mathcal{O}_2\rangle$ approach to the same value for different b.

fixed b is diverged as $T \to 0$, which implies that the backreaction on the bulk metric can not be ignored in this case. Moreover, we also find that the expectation values for both the condensate operators increase with b. And then, we fit these condensation curves for the operators $\mathcal{O}_1$ and $\mathcal{O}_2$ near $T \to T_c$ and present the results in the table (I). It is easy to find that the critical temperature for $\mathcal{O}_2$ decreases as b increases, while for $\mathcal{O}_1$ it first decreases and then increases. These properties are also shown in Fig.2 in which we re-plotted the condensation curves for the operators $\mathcal{O}_1$ and $\mathcal{O}_2$ by replacing T_c with T_0, where T_0 is the critical temperature at $b = 0$. From the Fig.2, it is interesting for us to find that the expectation values $\langle\mathcal{O}_2\rangle$ for different b tend to the same value at the very low temperature. But the expectation values $\langle\mathcal{O}_1\rangle$ do not possess this behavior. These differences could be explained by the fact that the operator $\mathcal{O}_2$ corresponds to a pair of fermions, while the operator $\mathcal{O}_1$ to a pair of bosons [5].

IV. THE ELECTRICAL CONDUCTIVITY

In this section we will compute the electrical conductivity by perturbing the Maxwell field. For simplicity, we adopt to the probe approximation and neglect the effect of the perturbation of background metric. Assuming that the perturbation of the vector potential has a form $\delta A_x = A_x(r)e^{-i\omega t}$ [5], we can obtain the linearized equation of motion

$$A_x'' + \frac{f'}{f}A_x' + \left(\frac{\omega^2}{f^2} - \frac{2q^2\psi^2}{f}\right)A_x = 0. \tag{20}$$

Since there exists only the ingoing wave at the black hole horizon, the boundary condition on A_x near $r \sim r_H$ in the Schwarzschild-AdS black hole spacetime with a global monopole can be expressed as

$$A_x = f^{-\frac{i\omega r_H}{(3r_H^2 - \bar{b})}} .$$
(21)

From Eqs. (16) and (20), it is easy to obtain that at the spatial infinity A_x possesses the asymptotic behavior

$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + ...$$
(22)

From the AdS/CFT, it is well known that $A_x^{(0)}$ and $A_x^{(1)}$ in the bulk corresponds to the source and the
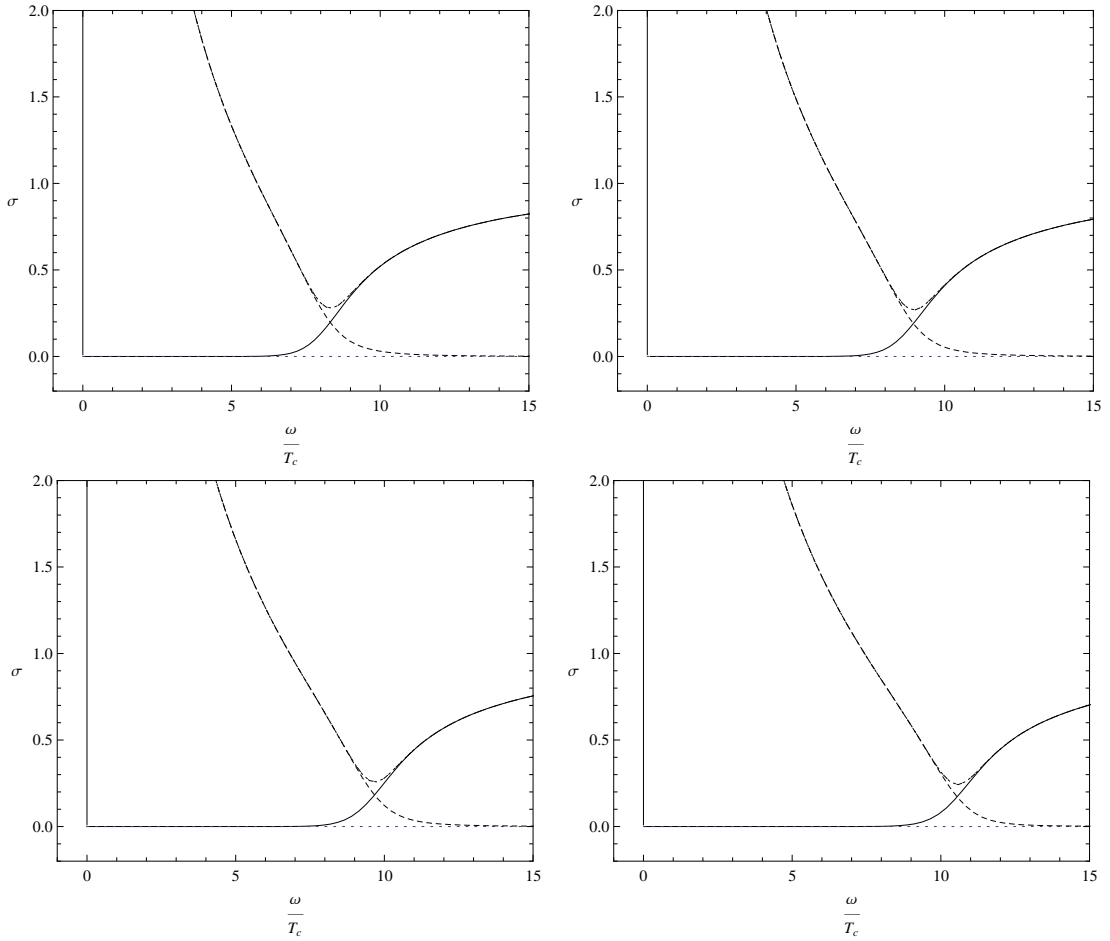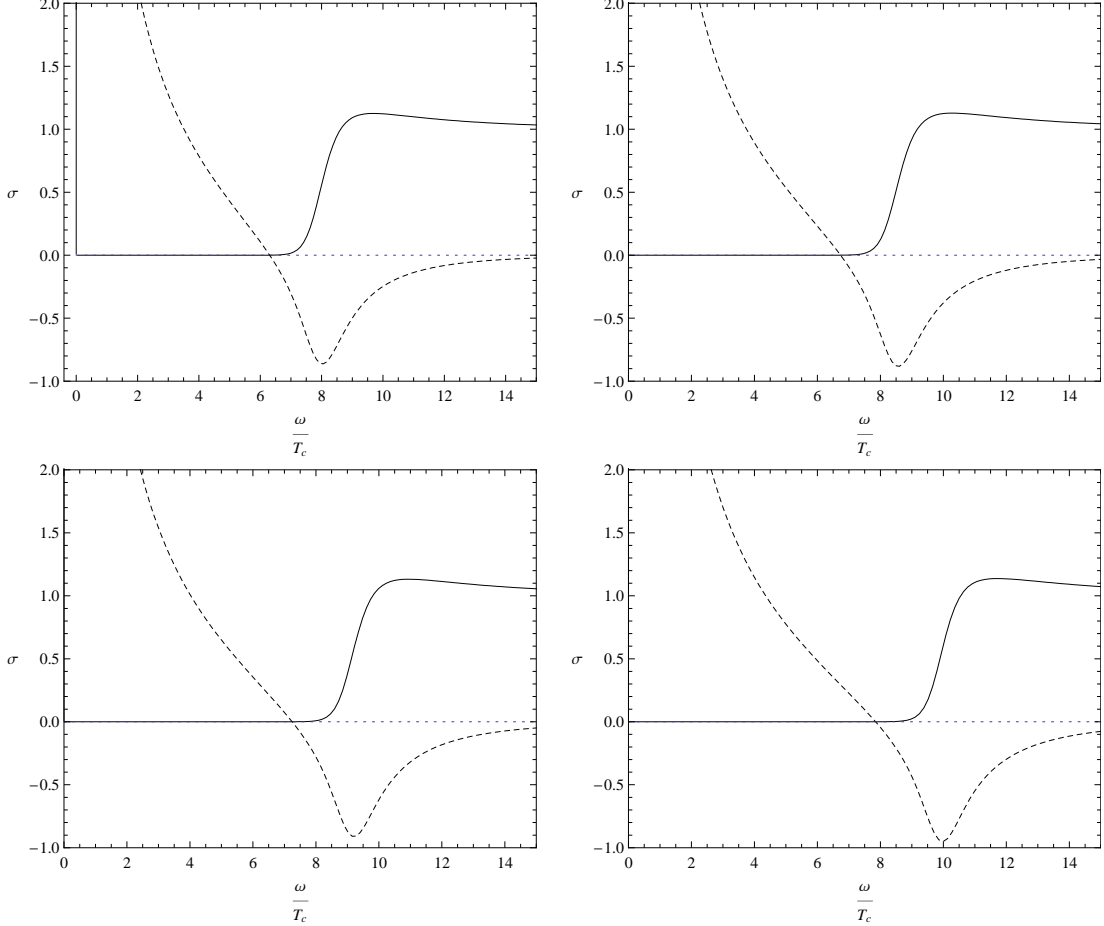

FIG. 3: The frequency dependent conductivity with different values of b for the operator $\langle \mathcal{O}_1 \rangle$. Each plot is at low temperatures, about $T/Tc \approx 0.20$. The up-left, up-right, bottom-left and bottom-right are corresponding to the cases $b = 0$, 0.25, 0.5 and $b = 0.75$, respectively. The solid, dashed and dash-dotted curves denote the real part $Re\sigma$, the imaginary part $Im\sigma$ and the module $|\sigma|$ of conductivity, respectively.

expectation value for the current on the CFT boundary, respectively. Thus, we have [5]

$$A_x = A_x^{(0)}, \qquad \langle J_x \rangle = A_x^{(1)}.$$
(23)

According to the Ohm's law, we can calculate the conductivity by

$$\sigma(\omega) = \frac{\langle J_x \rangle}{E_x} = -\frac{i\langle J_x \rangle}{\omega A_x} = -i\frac{A_x^{(1)}}{\omega A_x^{(0)}}. \tag{24}$$

In Figs. 3 and 4, we plot the frequency dependent conductivity with different b for the operators $\mathcal{O}_1$ at

FIG. 4: The frequency dependent conductivity with different values of b for the operator $\langle \mathcal{O}_2 \rangle$. Each plot is at low temperatures, about $T/Tc \approx 0.10$. The up-left, up-right, bottom-left and bottom-right are corresponding to the cases $b = 0$, 0.25, 0.5 and 0.75, respectively. The solid and dashed curves denote the real and imaginary parts of conductivity, respectively.

temperature $T/Tc \approx 0.20$ and $\mathcal{O}_2$ at $T/Tc \approx 0.10$, respectively. For the condensate operator $\mathcal{O}_1$, we have $\lambda = 1$, *i.e.*, $m^2 L^2 = -2$, which is above the Breitenlohner-Freedman bound [45]. We find that the real part increases and imaginary part of the conductivity decreases monotonously with the frequency ω for fixed b. Each modules of σ has a minimum value, which is the similar to that obtained in Ref. [46]. Moreover, from the figure (3), we also find that the value of $\frac{\omega_g}{T_c}$ increases with the increase of b. For the condensate operator $\mathcal{O}_2$ ($\lambda = 2$), we find from the figure (4) that, as the frequency ω increases for fixed b, the real part possesses the similar behavior as that in the case $\lambda = 1$. However, the imaginary part has a minimum value in this case

and the position of the minimum value moves along right when b increases . Moreover, as in other literatures [5, 6, 13, 20], we find that at $\omega = 0$ the real part of conductivity behaves as a delta function and the imaginary part exists a pole in the background with the global monopole. Similarly, this can be explained by using the Kramers-Kronig relation.

V.   SUMMARY

In this paper we studied the holographic superconductors in the Schwarzschild-AdS black-hole spacetime with a global monopole through a charged complex scalar field. We adopted to the probe approximation and solved the coupled nonlinear equations of the system for different b, which is related to the symmetry breaking scale η due to the presence of the global monopole. We found that the global monopole presents the different effects on the different condensates $\mathcal{O}_1$ and $\mathcal{O}_2$. As b increases, the critical temperature for $\mathcal{O}_2$ decreases, while for $\mathcal{O}_1$ it first decreases and then increases. Moreover, we also find that the expectation values $\langle\mathcal{O}_2\rangle$ for different b tend to the same value at very low temperature, while the expectation values for $\langle\mathcal{O}_1\rangle$ do not possess this behavior. These differences could be explained by the fact that the operator $\mathcal{O}_2$ corresponds to a pair of fermions and the operator $\mathcal{O}_1$ to a pair of bosons [5]. Moreover, we discussed the electric conductive at low temperature. Our results showed that the real part of the conductivity increases as the frequency ω increases for all b. For the case $\lambda = 1$ the imaginary part decreases monotonically with ω and the module of σ has a minimum value. The position of the minimum value is near $\frac{\omega_g}{T_c} \approx 8$ and moves along right with b. For $\lambda = 2$, the position of the minimum value of the imaginary part is similar to the position of the minimum value of module in the case $\lambda = 1$. These results can help us know more about holographic superconductors in the asymptotic AdS black holes.

Acknowledgments

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